Seasonal Adjustment of Time Series During the Pandemic

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Research and Methodology Directorate

Thanks to Kathy McDonald-Johnson

Economic Statistical Methods Division

United States® Census Bureau

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Question 1: How do we deal with the large outliers caused by the pandemic?

Question 2: For each time series, how do we tell when pandemic effects have ended, and seasonal adjustment procedures can go back to "normal"?

Question 3: To what extent does data from the pandemic effects period (March or April 2020 until when?) provide usable information about the seasonality of any given time series?

- Reinforcing info if the seasonal pattern has not changed, or
- Evidence for a change in seasonal pattern and to estimate the new pattern

About testing for a change in seasonal pattern:

- Mathematically, we can test for a pandemic induced change in seasonal pattern given 11 additional months of data (i.e., data through Feb or March of 2021).
  - This would assume all the unusual behavior starting with the pandemic is due to a change in seasonal pattern – which doesn't make sense.
- Practically speaking, we need some year(s) of data after the pandemic effects have ended to test for a change in seasonal pattern and to estimate it.

How X-11 seasonal adjustment is done (by X-13ARIMA-SEATS)

1. Fit a seasonal RegARIMA model to the (log) time series
   - Regression part of the model accounts for any calendar effects (trading-day and holiday), and for outliers (pre-specified or detected by the program)
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For ARIMA model-based seasonal adjustment one would replace X-11 by SEATS at Step 4.

Notes on X-11 seasonal adjustment

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- Can choose between additive (rare), multiplicative (common), or log-additive decompositions. (Log-additive is typically close to multiplicative.)
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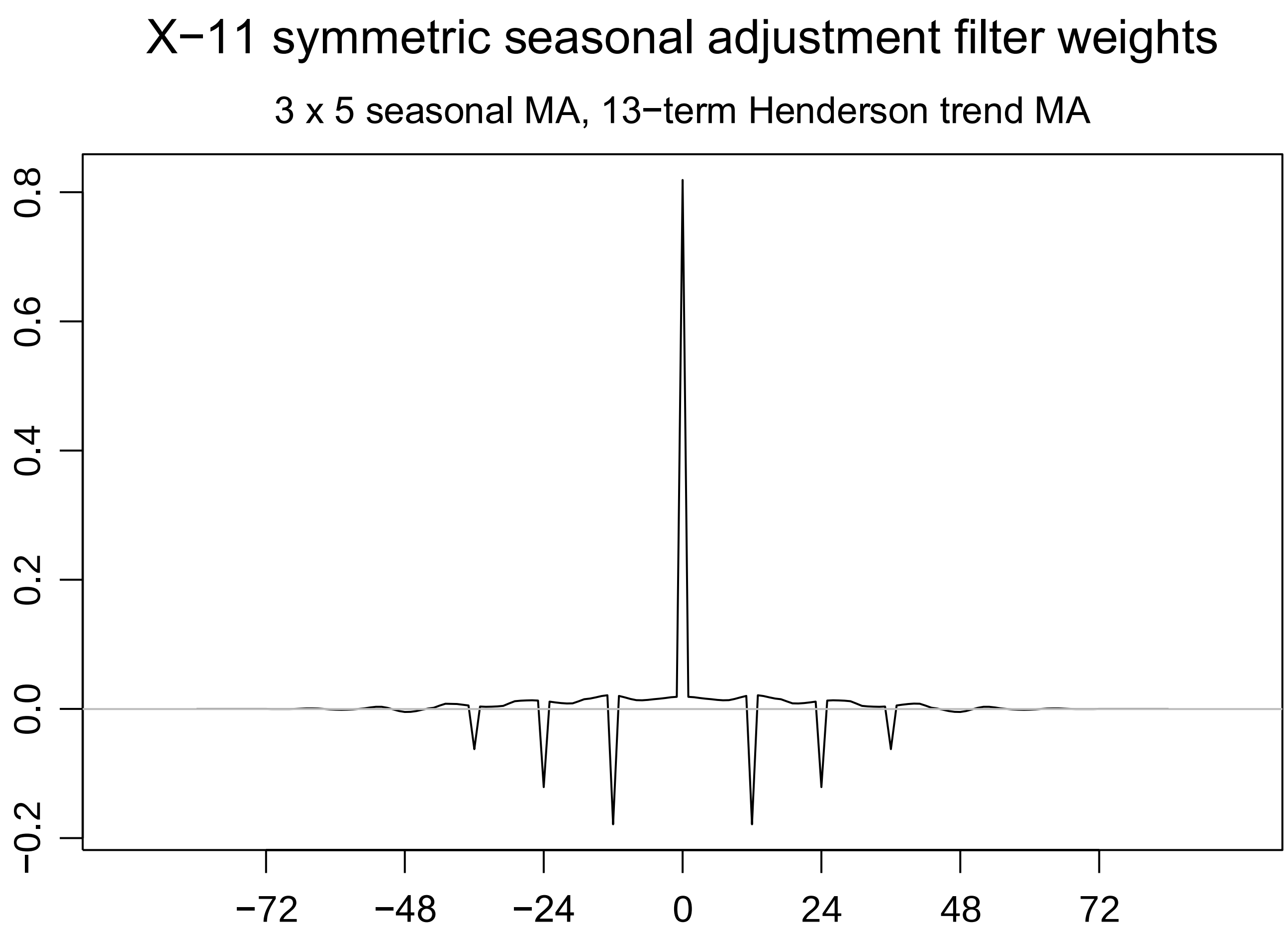
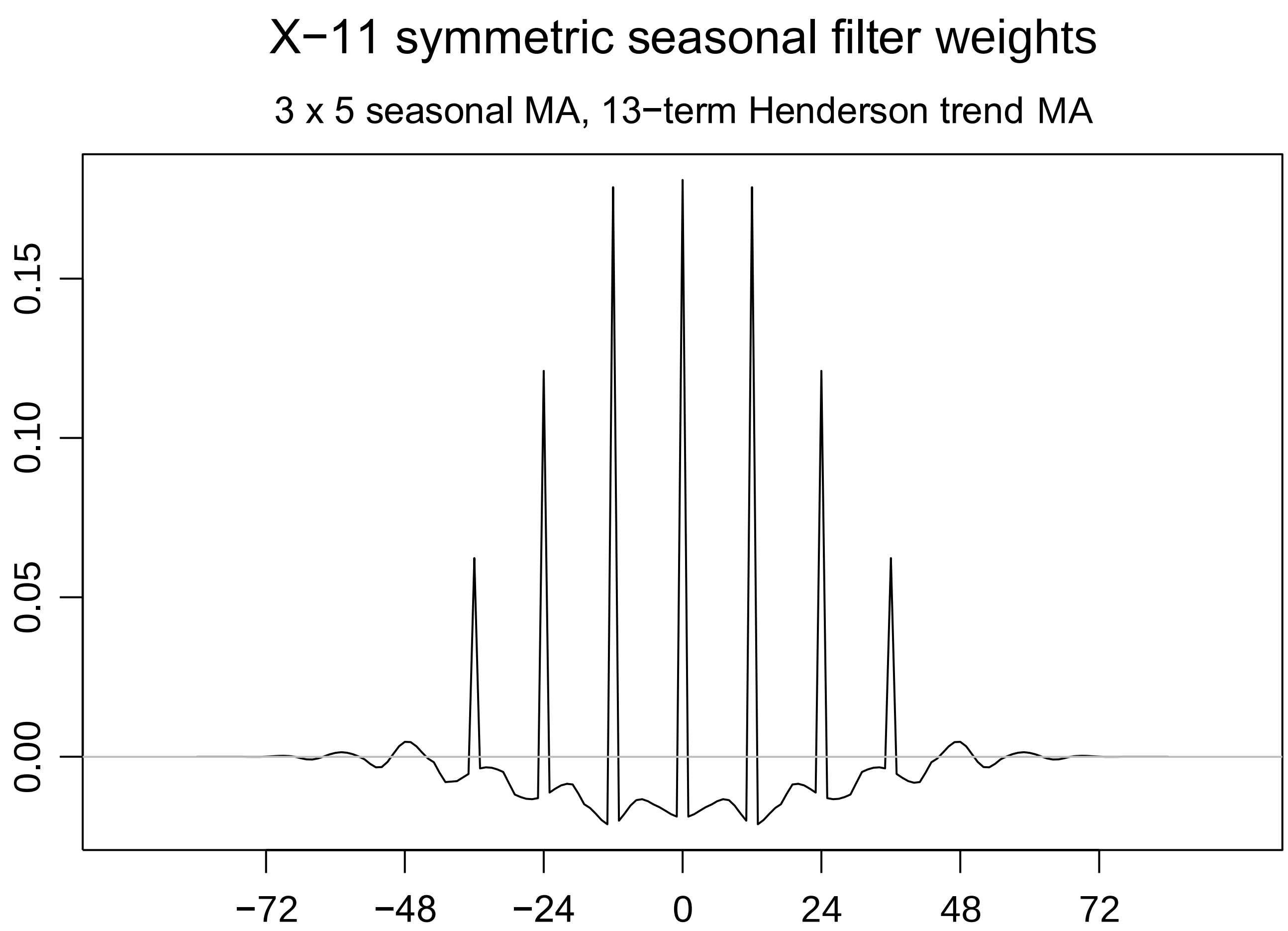
  - Asymmetric filters are used at the ends of the series
- ARIMA model-based adjustment also uses linear filters (derived from the estimated model) that can be very similar to those of X-11


X−11 symmetric seasonal adjustment filter weights
3 x 5 seasonal MA, 13−term Henderson trend MA
0.8
0.6
0.4
0.2
0.0
−0.2
−72
−48
−24
0
24
48
72


X−11 symmetric seasonal filter weights
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0.00
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Approach to dealing with pandemic effects in seasonal adjustment

- Use data prior to the pandemic period (up to February or March 2020) for estimating the seasonal factors
- Project (forecast) the seasonal factors ahead through the pandemic period
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This is accomplished via the X-13 program by specifying all pandemic period data to be outliers. Effectively, this turns the pandemic period into missing data. This is our answer to Question 1.

About Questions 2 and 3

Question 2: When do we turn off the pandemic outlier sequence?

- Some economic sectors are back to “normal.”
  - Outliers were no longer significant (even with a critical value of 2)
  - During last year's annual review, some previous pandemic outliers were removed from models because they were not significant.
- Quarterly services and monthly retail and wholesale trade still see pandemic effects.
  - Analysts provide valuable information from news sources or possibly company reports on how much each individual kind of business is still affected.

Question 3: Has the seasonal pattern changed post-pandemic?

- For some series, we are now approaching the point where we can consider trying to answer this question, but generally this probably will require more data.
- Note that X-11 and ARIMA model-based seasonal adjustment allow for evolving, though not for sudden, changes in seasonal patterns.

Alternative outlier forms allowed by X-13

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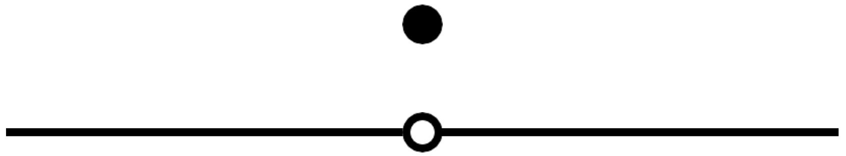
- For a run of outliers, results ***within the run*** will be the same whatever type of outlier is used
- The type of outlier used does matter once we get past the end of the run

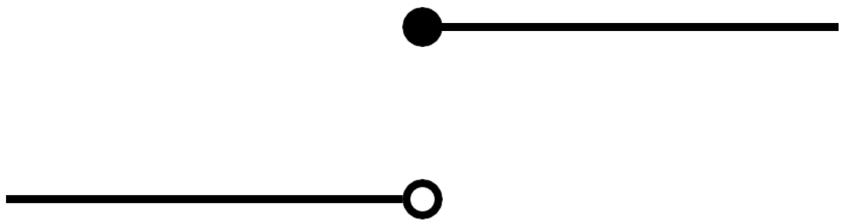
Additive Outlier (AO)



Regressor for AO at time t_0

$$\begin{cases} 1 \text{ for } t = t_0 \\ 0 \text{ for } t \neq t_0 \end{cases}$$

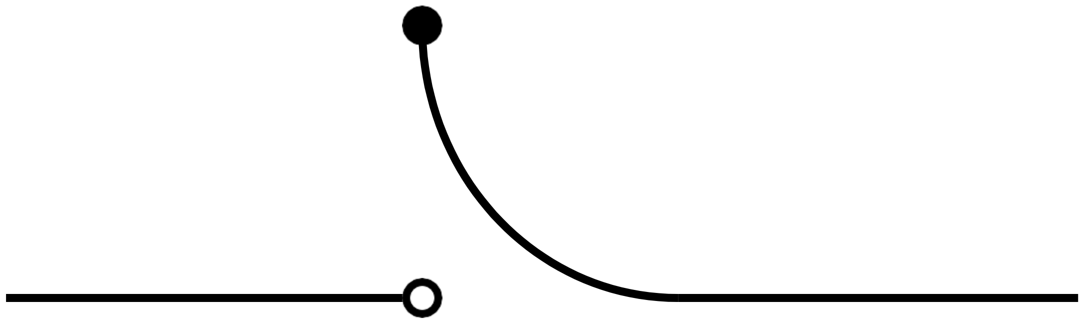
Level Shift (LS)



Regressor for LS at time t_0

$$\begin{cases} -1 & \text{for } t < t_0 \\ 0 & \text{for } t \geq t_0 \end{cases}$$

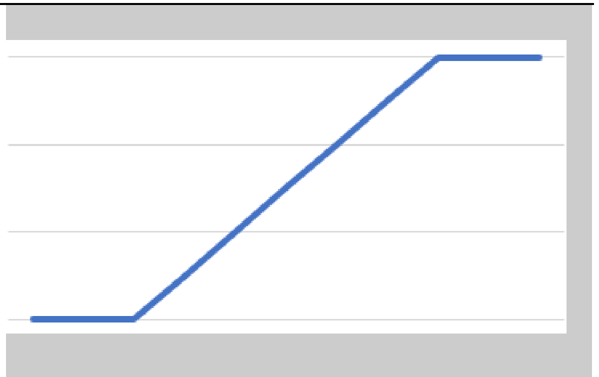
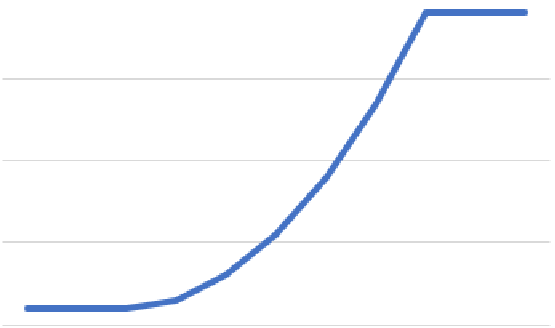
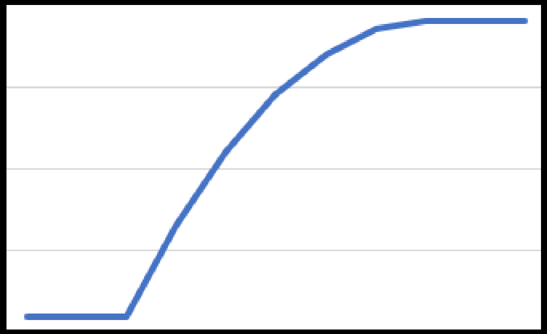
Temporary Change (TC)



Regressor for TC at t_0

$$\begin{cases} 0 & \text{for } t < t_0 \\ \alpha^{tt-tt_0} & \text{for } t \geq t_0 \end{cases}$$

where α is the rate of decay back to the previous level, $0 < \alpha < 1$ (default: 0.7 for monthly and 0.343 for quarterly time series)

Ramp Effect	Regression Variable	Graph of 6-Month Increase
Linear Ramp	$RRRR_{tt}^{(tt0,tt1)} = \begin{cases} tt_0 - tt_1 & ffffff\ tt \le tt_0 \\ tt - tt_1 & ffffff\ tt_0 < tt < tt_1 \\ 0 & ffffff\ tt \ge tt_1 \end{cases}$	
Increasing Quadratic Ramp	$\cap\cap\cap_{tt}^{(tt0,tt1)} = \begin{cases} -(tt_1 - tt_0)^2 & ffffff\ tt \le tt_0 \\ (tt - tt_0)^2 - (tt_1 - tt_0)^2 & ffffff\ tt_0 < tt < tt_1 \\ 0 & ffffff\ tt \ge tt_1 \end{cases}$	
Decreasing Quadratic Ramp	$\cap\cap\cap\cap_{tt}^{(tt0,tt1)} = \begin{cases} -(tt_1 - tt_0)^2 & ffffff\ tt \le tt_0 \\ -(tt_1 - tt)^2 & ffffff\ tt_0 < tt < tt_1 \\ 0 & ffffff\ tt \ge tt_1 \end{cases}$	

Outlier detection approach of X-13 (taken from X-12 and earlier software)

RegARIMA model with an AO at time kk:

$$yy_{tt} = xx'_{tt}\beta\beta + \omega\omega \times AAAA_{tt}^{(kk)} + zz_{tt} \qquad tt = 1, \ldots, nn \qquad (*)$$

zz_{tt} follows an ARIMA model

AO detection:

1. Estimate RegARIMA model (*), separately for each $kk = 1, \ldots, nn$.
2. Save t-statistics $\lambda_{kk} = �_{kk} / sstt\,ss\,ssdddd(�_{kk})$ for $kk = 1, \ldots, nn$.
3. Compare $\lambda_{mmmmmm} = \max_{kk} |\lambda_{kk}|$ to a critical value cc. If $\lambda_{mmmmmm} > cc$ add $AAAA_{tt}^{(kk)}$ to the xx'_{tt} in (*).
4. Continue until no more outliers are detected. (There is also a backward deletion at the end.)

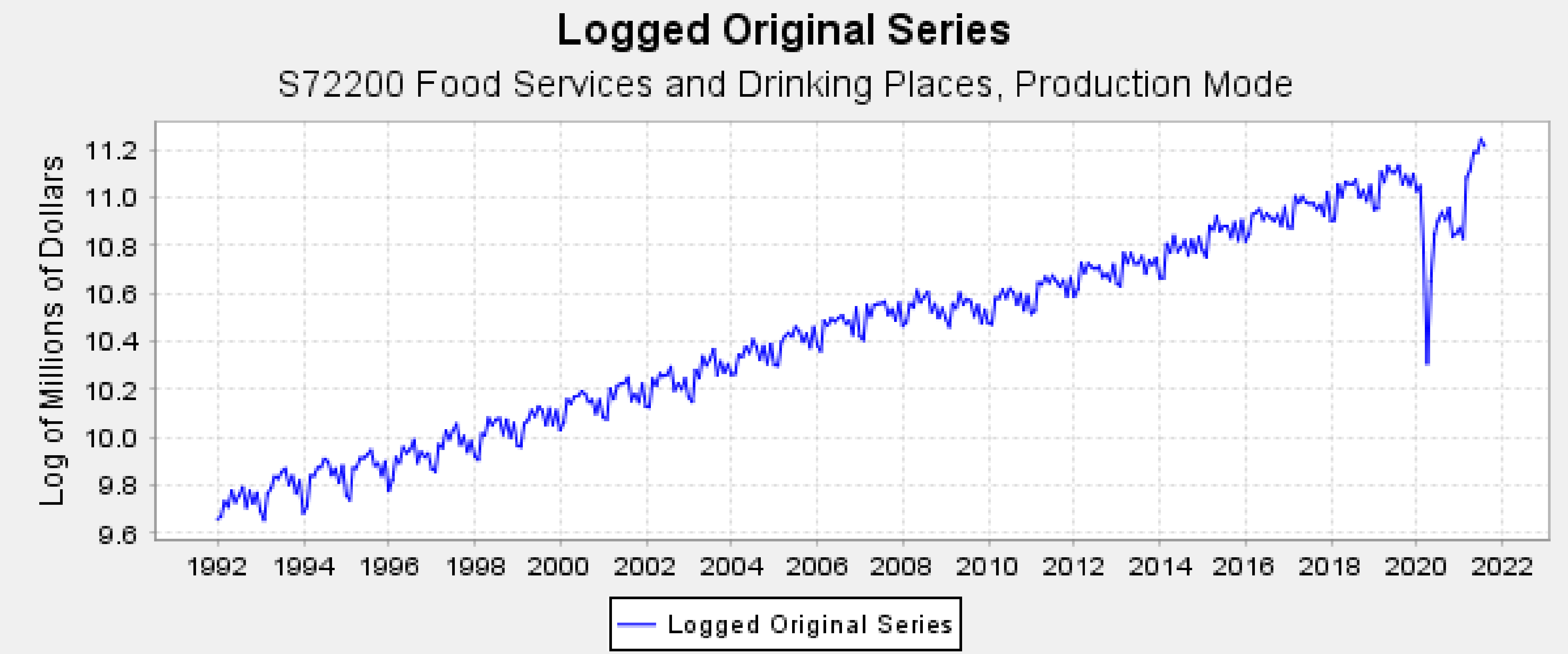
Notes:

- cc comes from an asymptotic result that depends on nn and accounts for searching the series for outliers. For a 5% test with $nn \approx 150$ or larger, $cc \approx 4$. Compare this to $cc \approx 2$ with no searching.
- We can use the same scheme to test for LSs and TCs, separately or in combination. Default is to test for AOs and LSs.
- Known outliers can be specified in xx'_{tt} of the initial model (intervention analysis).

Examples From Monthly Retail Trade and Food Services, U.S. Census Bureau

- Estimates are from surveys and are subject to sampling and nonsampling error
- [Information about the data collection and estimation is online at census.gov/retail/how_surveys_are_collected.html](census.gov/retail/how_surveys_are_collected.html)

Food Services and Drinking Places, Sales, Log Scale (Millions of Dollars), From 1992



Source: [Monthly Retail Trade and Food Services, U.S. Census Bureau (census.gov/retail/)](census.gov/retail/)

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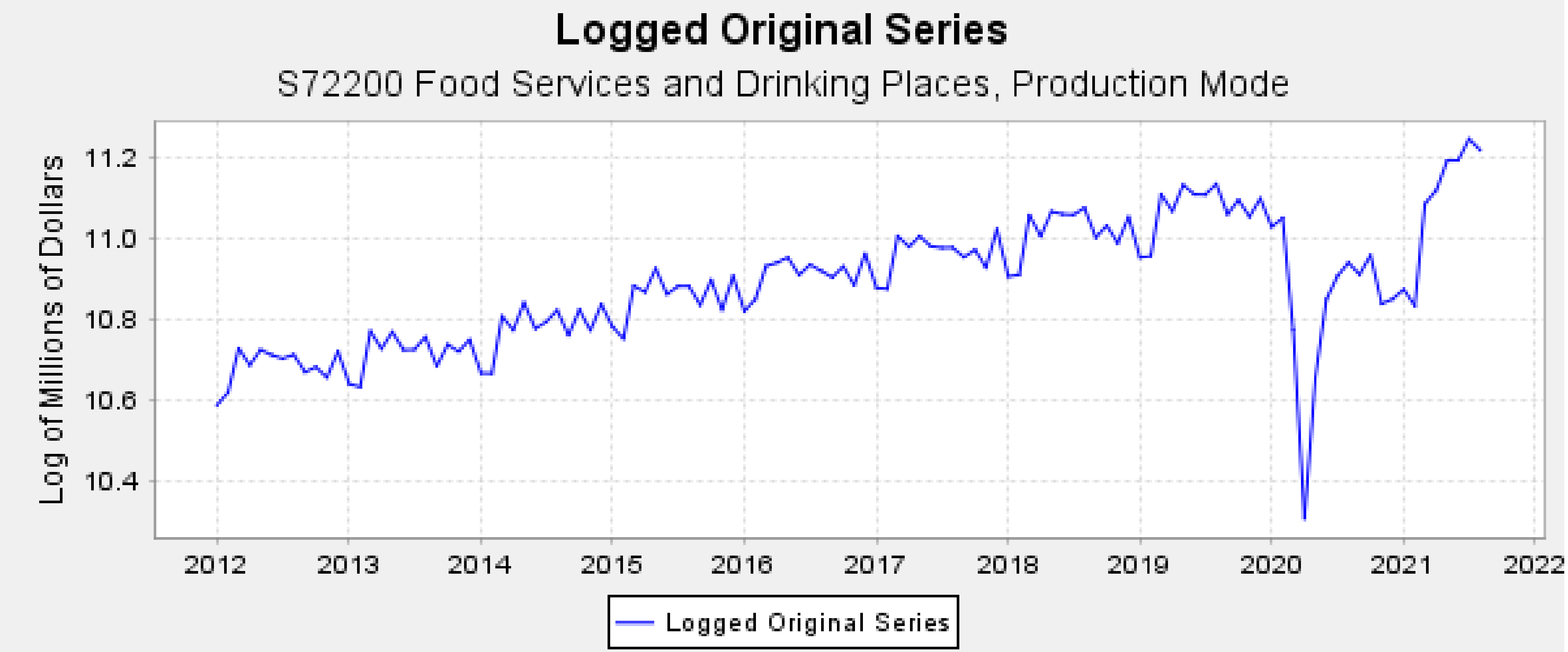
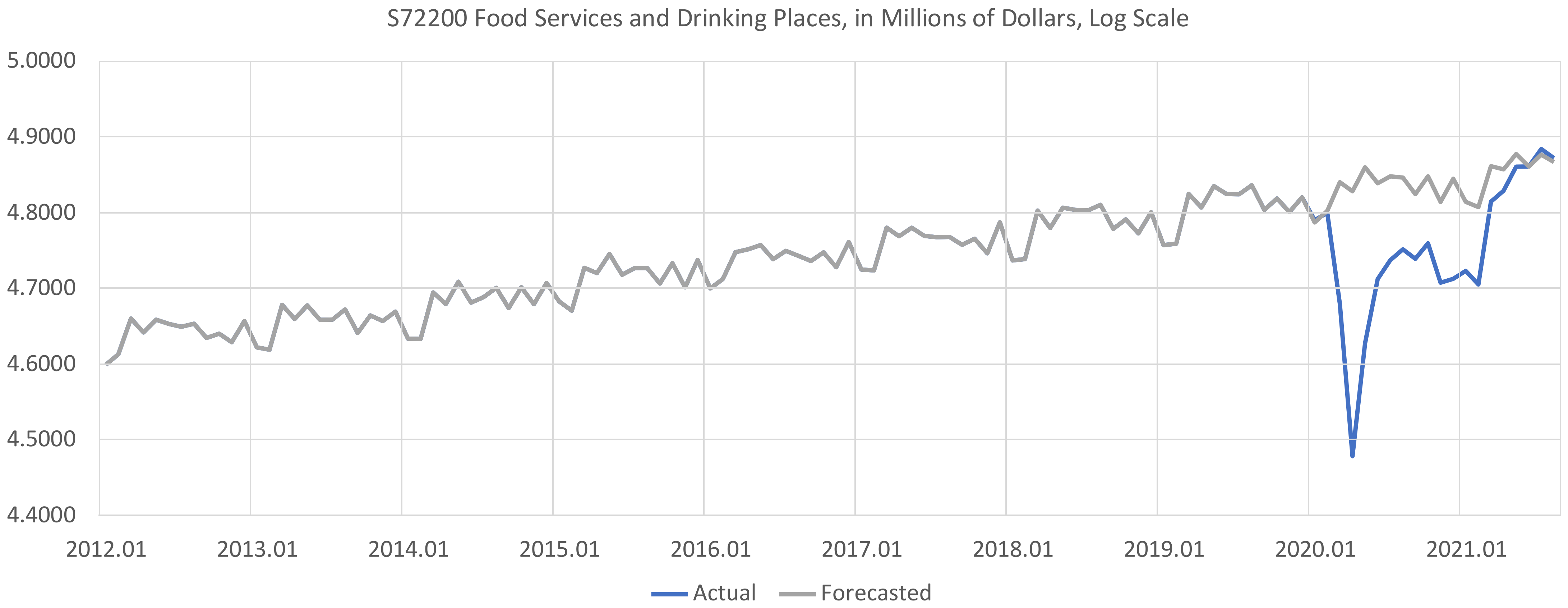


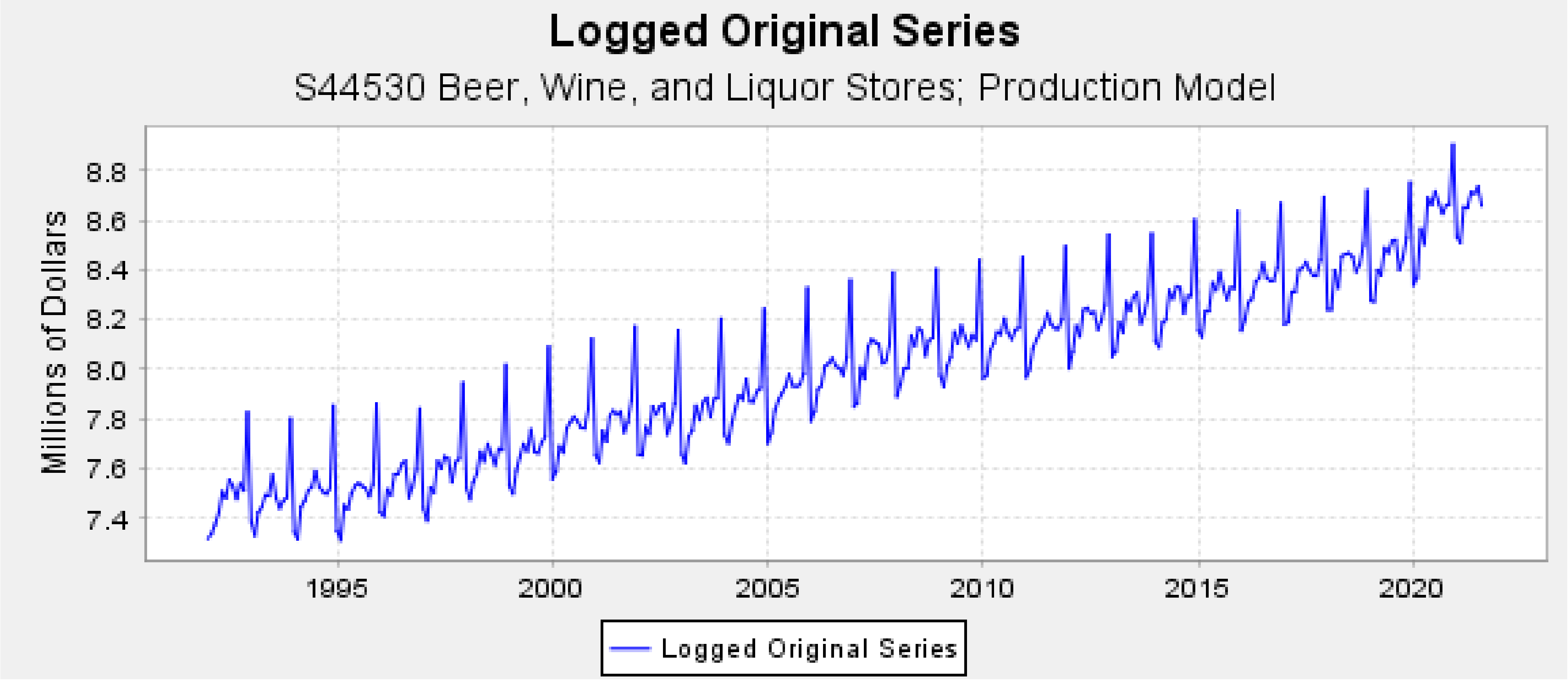
Source: [Monthly Retail Trade and Food Services, U.S. Census Bureau (census.gov/retail/)](census.gov/retail/)

Food Services and Drinking Places, Sales, Log Scale (Millions of Dollars), From 2012, Forecasts vs. Actual



Source: [Monthly Retail Trade and Food Services, U.S. Census Bureau (census.gov/retail/)](https://www.census.gov/retail/)

Beer, Wine, and Liquor Store Sales, Log Scale (Millions of Dollars), From 1992



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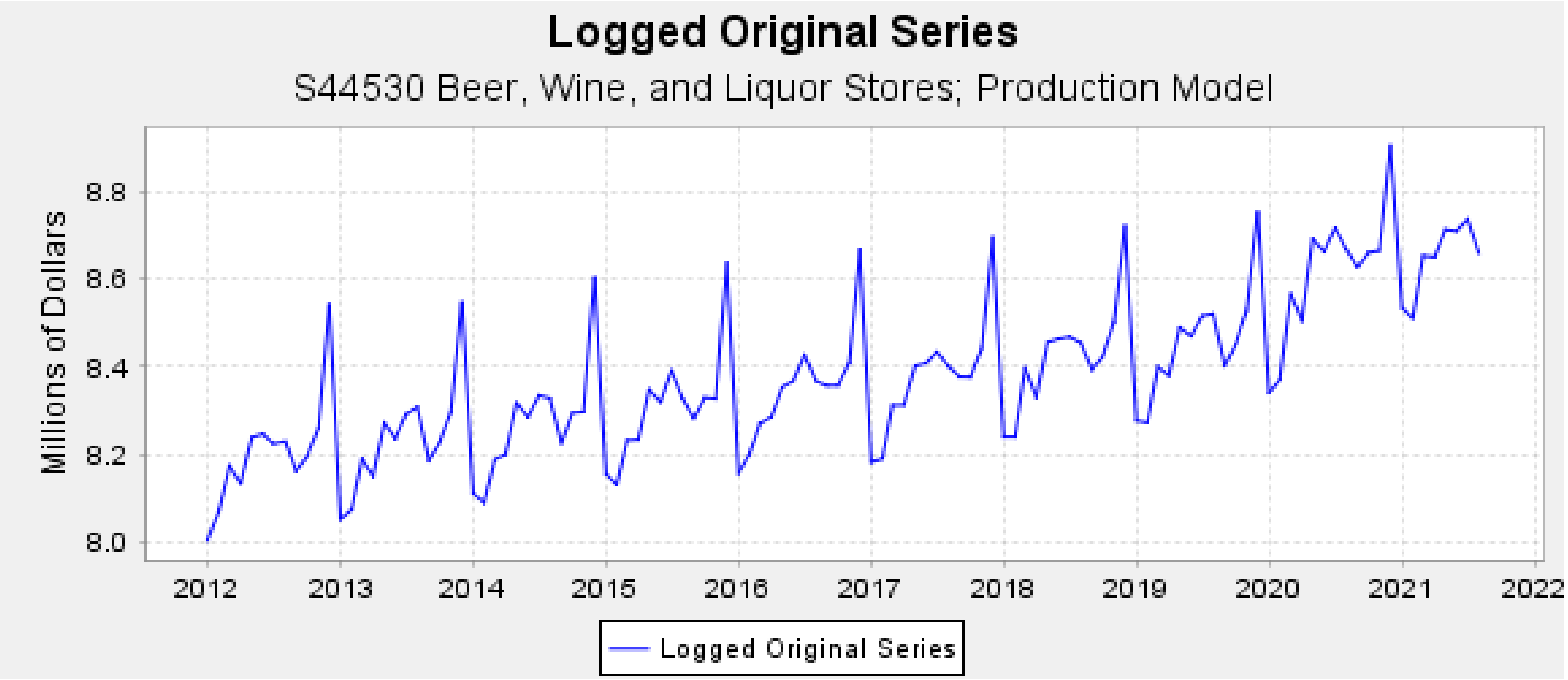

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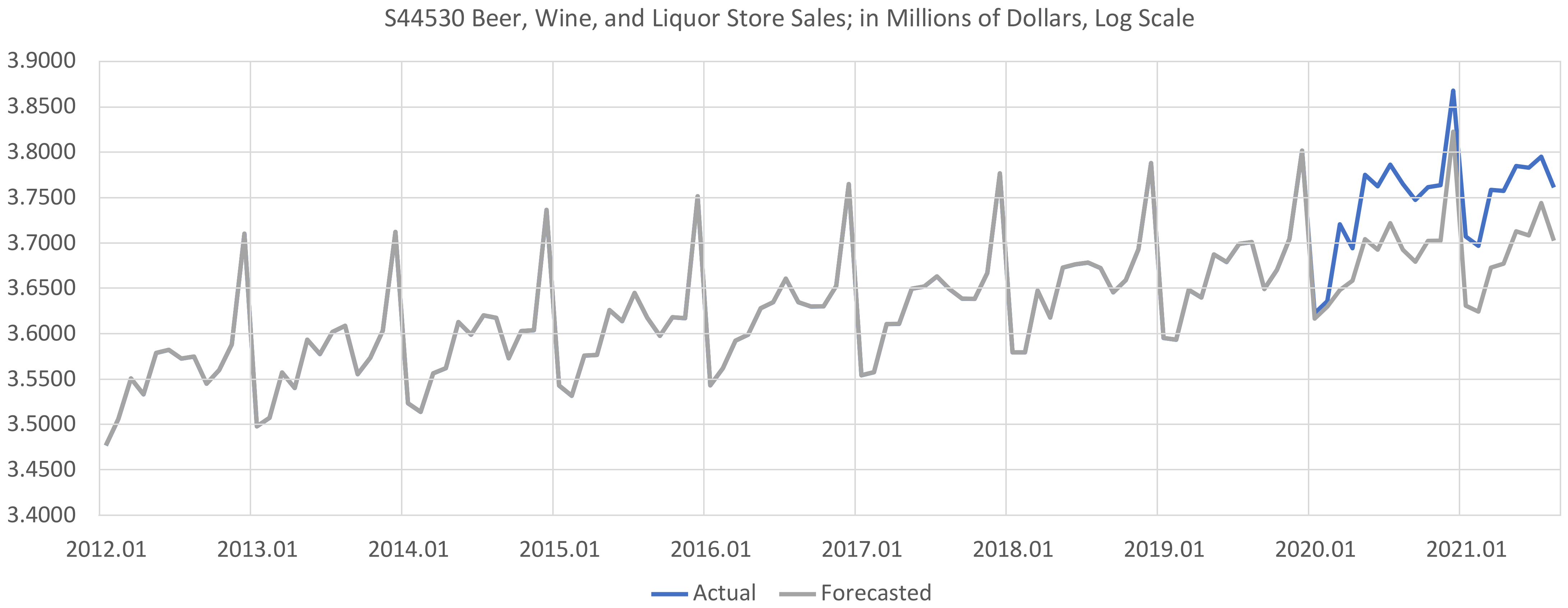
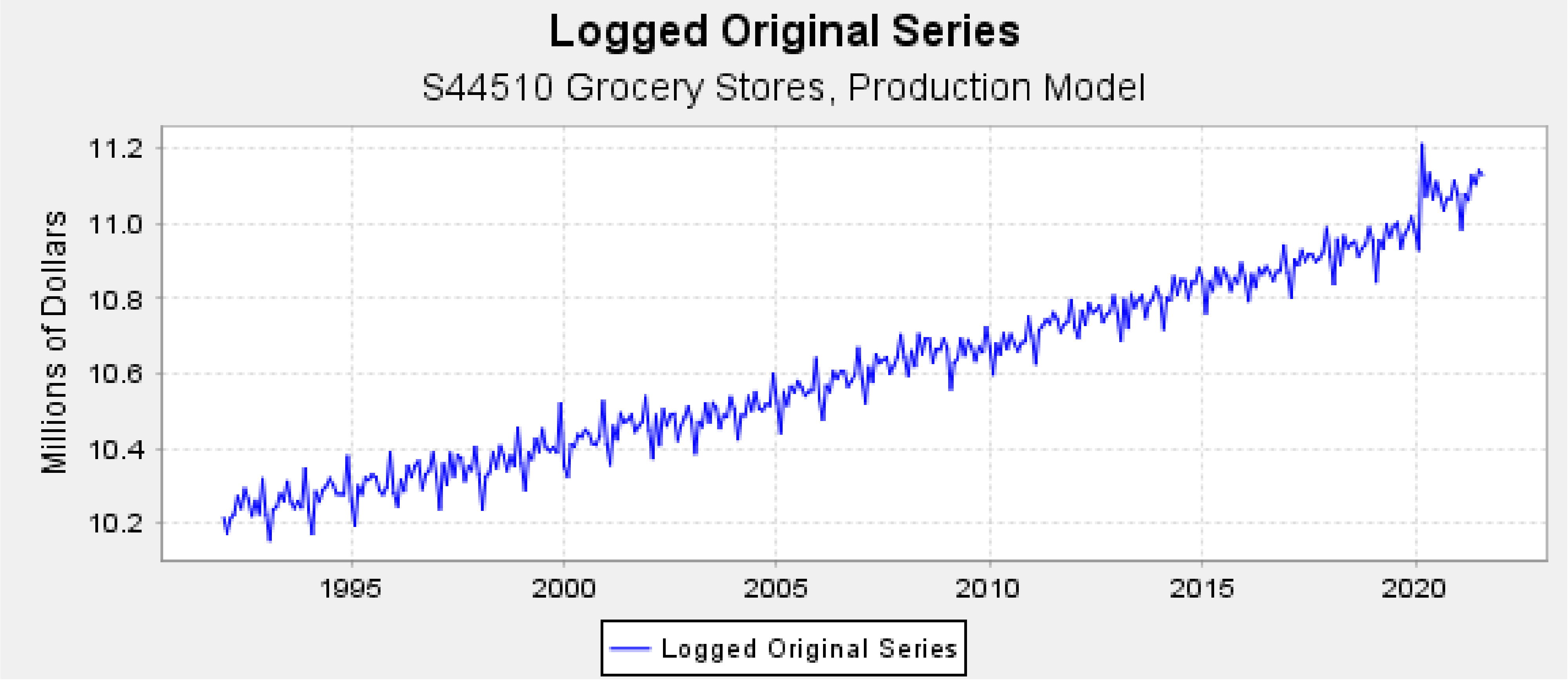


Source: [Monthly Retail Trade and Food Services, U.S. Census Bureau (census.gov/retail/)]

Scenes From a Grocery Store, March 2020

- Photos are courtesy of Suzanne Dorinski, U.S. Census Bureau

Grocery Store Sales, Log Scale (Millions of Dollars), From 1992



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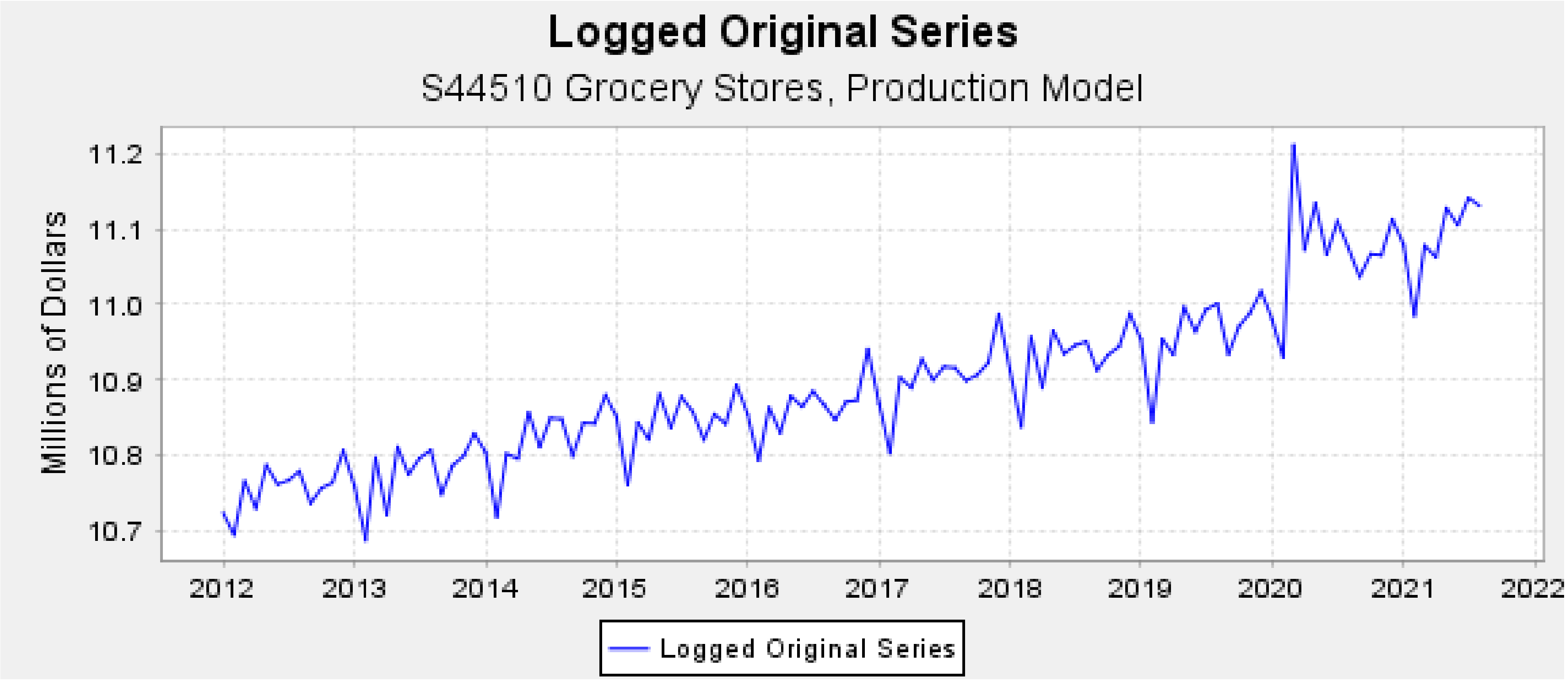

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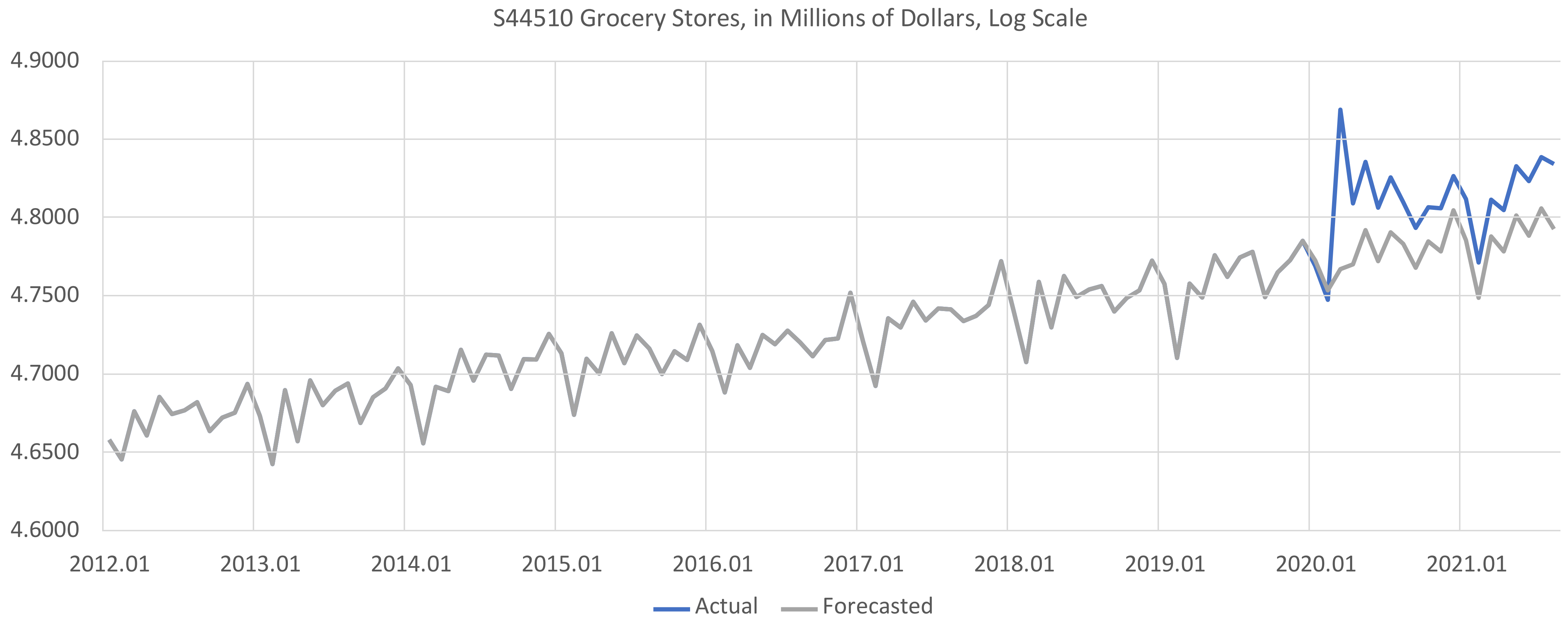
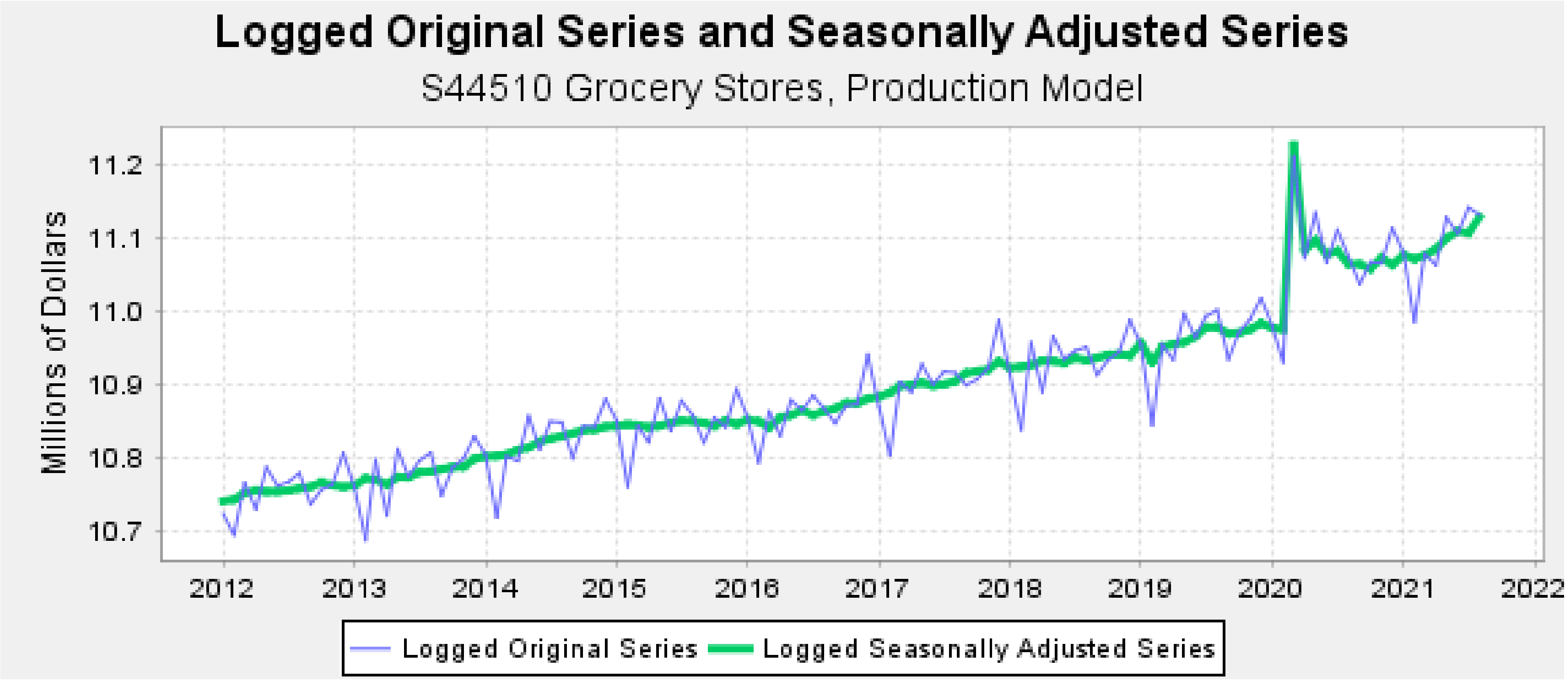


Source: [Monthly Retail Trade and Food Services, U.S. Census Bureau (census.gov/retail/)](census.gov/retail/)

Grocery Store Sales, Log Scale (Millions of Dollars), From 2012, Seasonally Adjusted Series



Source: [Monthly Retail Trade and Food Services, U.S. Census Bureau (census.gov/retail/)](census.gov/retail/)

Rephrasing our 3 Questions (for the committee)

Question 1: Do you have any general thoughts about how we are dealing with pandemic effects in seasonal adjustment?

Question 2: Do you have any ideas for determining when pandemic effects have ended, realizing that this can vary across and within economic sectors?

Question 3: Do you have any ideas for determining whether seasonal patterns have changed (shifted) post pandemic?

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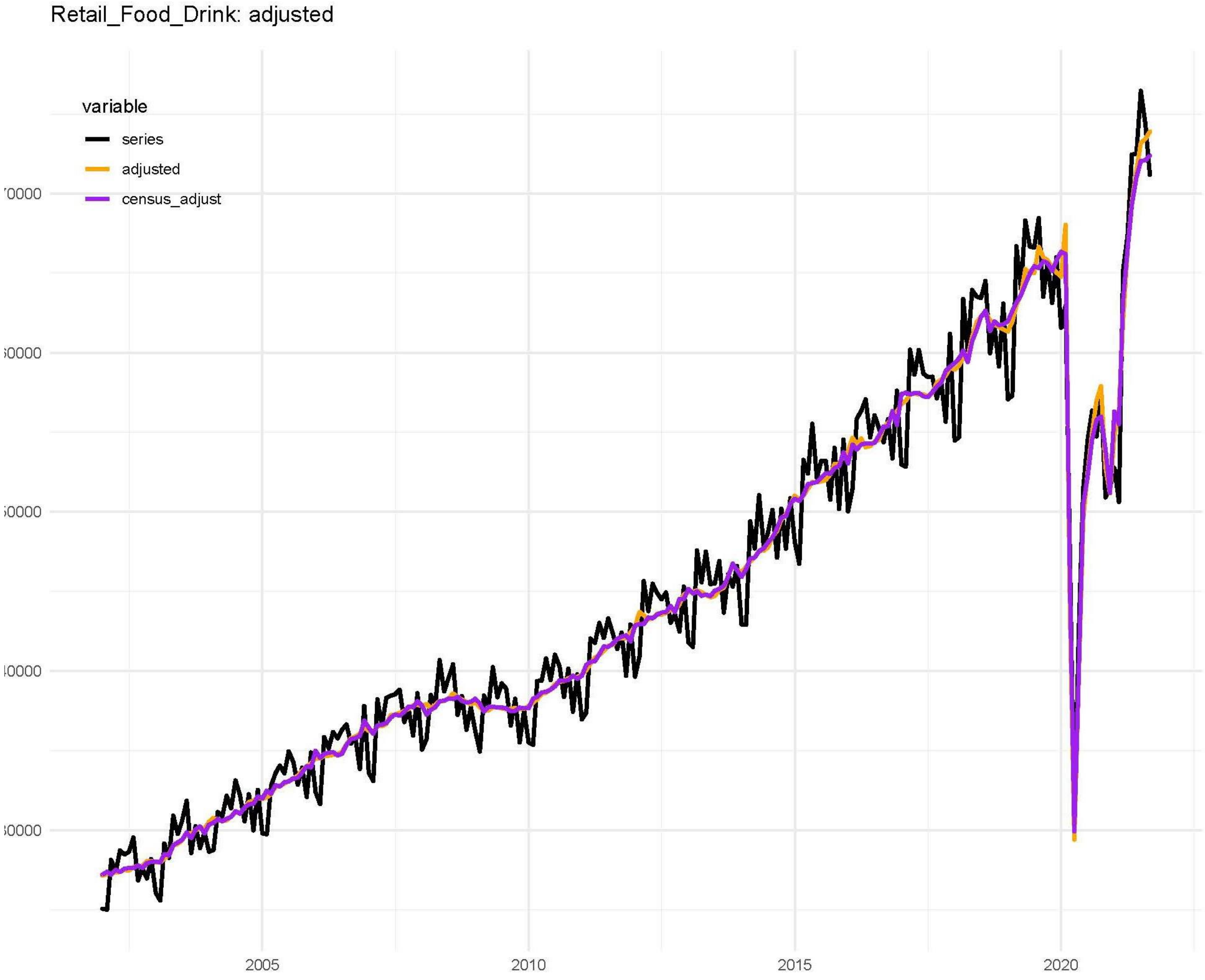
Two Experimental Methodologies

- "A Bayesian Framework for Modeling Extreme Events and Their Impact on Time Series in the Post-Covid-19 Era," by Anindya Roy and Tucker McElroy.
  - Models AO, LS, TC effects as stochastic processes
  - Bayesian estimation quantifies uncertainty in crisis epoch identification
- "Analysis of Crisis Effects via Maximum Entropy Shrinkage," by Tucker McElroy.
  - Defines crisis effects (AO and LS) such that Gaussian entropy is increased with their removal
  - Method allows for shrinkage as opposed to either/or approach to extremes

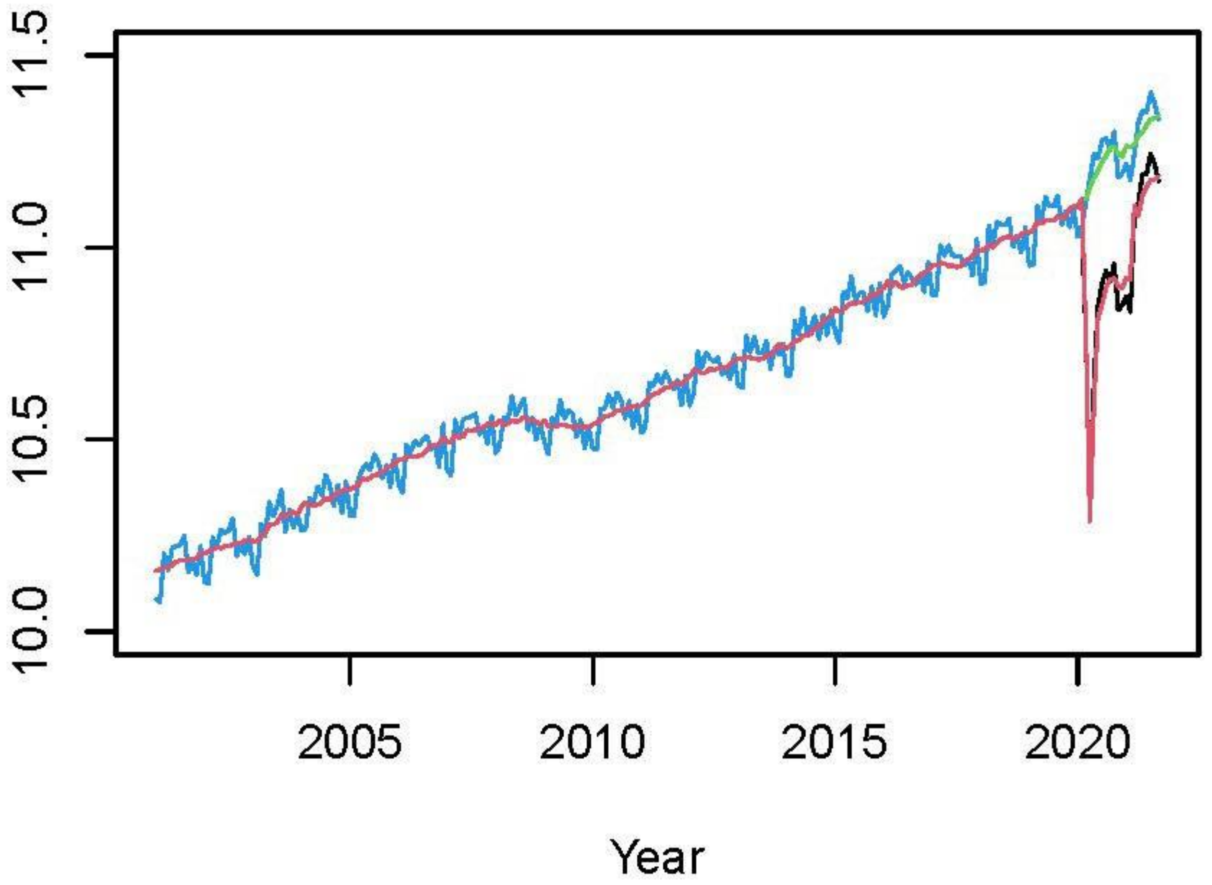
Bayesian Method

- Example is “Food Services and Drinking Places,” monthly 2001.Jan-2021.Sep (U.S. Census Bureau).
- Original time series (black), Bayesian seasonally adjusted series (yellow), and X-13ARIMA-SEATS seasonally adjusted series (purple).
- Note: AO, LS, and TC effects are non-seasonal, hence are present in seasonally adjusted series.



Maximum Entropy Method

- Same data example, but modeled in log scale.
- Original time series (black), regularized data with outliers removed (blue), de-seasonalized regularized data (green), and final seasonally adjusted series (red).
- This method requires analyst to identify AO and LS epochs; but a bit faster/easier modeling than Bayesian approach.



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